Structural Invariance and Domain-Selective Response Under Fundamental Constant Deformation

An Empirical Investigation Across Four Fundamental Constants and Seven Physical Domains Using the Alignment Matrix Protocol

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Abstract

We report the results of a systematic, preregistered investigation into whether the variation of fundamental physical constants constitutes *structural activity* — changing the admissibility regime, state, or violation status of physical spectral ladders — or merely *metric activity* that rescales energy values while leaving ordering architecture intact.

Four constants are treated as deformation operators acting on physical ladders: the fine-structure constant α (Column I), the proton-to-electron mass ratio μ (Column II), the strong coupling constant α_s (Column III), and the gravitational coupling constant α_G (Column IV). Each constant is varied across a 17-point grid $\beta \in [0.80, 1.20]$ relative to its physical value, and the admissibility inequality

$$\mathrm{inv}(P_\varepsilon;\, L) \;\leq\; \nu\big(V_\varepsilon(L)\big)$$

is evaluated using the STRUC-I v1.0.4 computational chamber.

The corpus spans seven domain types: atomic spectra (H, He, Na, Li), molecular vibration-rotation spectra (CO, H_2, N_2, HCl, HD), cosmic microwave background (Planck 2018 TT/TE/EE), large-scale structure (DESI), planetary geoid gravity harmonics (Earth, Mars, Moon), nuclear level schemes (15 isotopes, ENSDF), and hadronic spectra (charmonium J/ψ family, PDG).

The main findings are as follows. (*i*) The USL inequality is not falsified at the physical value of any constant in any domain. (*ii*) Structural response is domain-selective and constant-selective: α is structurally active in the geoid (proxy) and sodium atomic domains but inactive in cosmology and hydrogen; μ produces a confirmed Type III-Max signal in H_2 (physical mass ratio $\beta^* \approx 1.00$) but is inactive in CO, N_2, and HCl; α_s and α_G produce no detectable structural signal across all tested systems. (*iii*) The Column I geoid result — a sharp admissibility minimum at $\alpha = 1$ attributed to the proxy deformation protocol — is refuted by Column IV: the physically grounded α_G deformation yields Type I across all three planetary bodies, with the true structural extremum γ^* lying below the sweep floor for each body. (*iv*) ^{48}Ca undergoes a dramatic cross-constant reversal: 17/17 marginal violations under α but complete inertness under α_s, demonstrating that constant-sensitivity is coupling-mechanism-specific rather than a property of the gap geometry alone. (*v*) ^{208}Pb returns perfect admissibility ($A_\kappa = 1.000$) at every tested deformation point under both α and α_s, establishing cross-constant inertness for doubly-magic nuclear structure at maximum shell closure. (*vi*) HD under inferred Tier A vibrational/rotational decomposition returns Type I throughout; apparent hard violations in an earlier approximate-decomposition run are identified as decomposition artifacts, demonstrating that molecular ladder classification is acutely sensitive to the quality of the energy-component separation.

Keywords: structural admissibility, fundamental constants, deformation operators, alignment matrix, admissibility inequality, UNNS substrate, fine-structure constant, proton-to-electron

mass ratio, strong coupling, gravitational coupling, molecular spectra, nuclear spectra, geoid harmonics, STRUC-I, Tier A decomposition, cross-constant analysis

Contents

1 Introduction

1.1 Motivation

Physical systems support ordered structures at every scale of organisation. Nuclear energy levels define reproducible excitation hierarchies; molecular rovibrational states arrange themselves in regular ladders; planetary gravity fields exhibit structured harmonic spectra; the cosmic microwave background carries the imprint of primordial acoustic oscillations. These orderings are not only empirically stable under physical perturbations — they also appear to satisfy a universal structural constraint [1, 2]: the number of ordering inversions induced by a bounded perturbation never exceeds the vulnerability capacity of the structure's gap geometry.

The present work asks a complementary question. Rather than asking whether ordered physical systems satisfy the admissibility inequality at their *actual* constant values, we ask what happens when the fundamental constants themselves are treated as deformation operators and varied systematically. Does varying α, μ, α_s, or α_G change the structural character of physical ladders? Does the admissibility inequality fail at some non-physical constant value? And, crucially, does the physical constant value occupy a structurally special position — a minimum, maximum, or transition point — in the resulting structural pressure landscape?

1.2 The Admissibility Framework

Let $L = (\ell_1 < \ell_2 < \cdots < \ell_n)$ be an ordered ladder. The gap sequence is $g_i = \ell_{i+1} - \ell_i > 0$. A perturbation family P_ε is a collection of bounded additive perturbations; the inversion count $\mathrm{inv}(P_\varepsilon; L)$ measures the worst-case number of order reversals the family induces on L. The vulnerability graph $V_\varepsilon(L)$ has one node per gap g_i and edges between gaps that are simultaneously invertible by a perturbation of scale ε. The vulnerability capacity $\nu(V_\varepsilon(L))$ is the size of the maximum independent set of this graph.

The admissibility inequality is:

$$\mathrm{inv}\big(P_\varepsilon;\, L\big) \;\le\; \nu\big(V_\varepsilon(L)\big). \tag{1}$$

The structural pressure ratio $\rho = \langle \mathrm{inv} \rangle / \nu$ measures how deeply a ladder uses its vulnerability budget. The admissibility rate

$$A_\kappa \;=\; \frac{1}{|\mathcal{K}|} \sum_{\kappa \in \mathcal{K}} \mathbf{1}\big[\mathrm{inv}(P_\kappa; L) \le \nu(V_\kappa(L))\big]$$

is the fraction of perturbation scales κ at which the inequality is satisfied. A clean violation of (1) is defined as $A_\kappa < 0.52$ in the STRUC-I v1.0.4 operationalisation.

1.3 The Alignment Matrix

The central organisational device of this investigation is the *Structural Response Atlas*, or *Alignment Matrix*: a table whose rows are physical systems and whose columns are fundamental constants, with each cell recording the structural response profile (System, Constant, Deformation Protocol) $\to$ Classification.

Columns are indexed I through IV in the order they were completed:

Column I Fine-structure constant α; proxy deformation; 5 domains; completed.

Column II Proton-to-electron mass ratio μ; Tier A vib/rot decomposition; molecular domain; completed.

Column III Strong coupling constant α_s; nuclear + hadronic domains; completed (null result).

Column IV Gravitational coupling constant α_G; planetary geoid; completed (null result, Column I proxy refuted).

1.4 Paper Organisation

Section 2 describes the STRUC-I chamber, the deformation protocols, and the classification schema. Sections 3–6 present results for each column. Section 7 synthesises the cross-constant findings. Section 8 discusses the universal admissibility status. Section 9 places the results in the broader context of the UNNS Substrate programme. Section 10 concludes.

2 Methods

2.1 STRUC-I v1.0.4 Chamber

The STRUC-I v1.0.4 chamber is a browser-based single-page application implementing a deterministic linear rank-order perturbation algorithm. For each ladder L at perturbation scale κ:

(i) The perturbation magnitude is $\varepsilon = \kappa \cdot \mathrm{median}(g_i)$.

(ii) A deterministic perturbation sequence is applied to the n elements of L.

(iii) Inversions and the vulnerability graph are computed analytically; no Monte Carlo sampling is used.

(iv) The inequality (1) is checked at each of 40 κ-steps log-spaced in $[0.01, 1.0]$.

(v) Output: per-ladder fingerprint including $\bar{\rho}$, $\max \rho$, A_κ, regime, and state.

Structural regimes and states. The STRUC-I output assigns each ladder to one of four states based on $\bar{\rho}$ and A_κ: Stable Structure ($\bar{\rho} < 0.30$, $A_\kappa = 1$); Weak Persistence ($0.30 \leq \bar{\rho} < 0.55$, $A_\kappa = 1$); Boundary-Stabilized ($\bar{\rho} \geq 0.55$, $A_\kappa \approx 1$); Transitional Structure (marginal $A_\kappa < 1$).

2.2 The Deformation Protocol Framework

A deformation protocol specifies how a fundamental constant c with physical value c_0 acts on a ladder L. We write $\beta = c/c_0$ for the relative deformation parameter and require β to act *non-uniformly* across the energy components of L; uniform scaling preserves all gap ratios and is structurally invisible.

Definition 1 (Tier A decomposition)**.** A deformation is *Tier A* if the energy components that scale differently with β are explicitly and physically derived from first principles or from locked published constants, with no approximation. A *proxy deformation* applies a heuristic non-uniform scaling in the absence of a first-principles coupling derivation. Only Tier A entries are eligible for insertion into the Alignment Matrix; proxy entries are logged with the caveat label PROXY.

2.3 Classification Schema

Structural response classifications follow the taxonomy of `MU_DEFORMATION_PROTOCOL_v2`:

- **Type I (Invariant):** $\Delta\bar{\rho} \leq 3\rho_{\mathrm{floor}}$; no regime or state change; no coherent trend.
- **Type II (Deformable):** $\Delta\bar{\rho} > 3\rho_{\mathrm{floor}}$; no structurally distinguished β-point.

- **Type III-Max (Anti-alignment):** Unique structural maximum at β^*; physical constant at a pressure peak.
- **Type III-Min (Alignment):** Unique structural minimum at β^*; physical constant at a pressure trough.
- **Type III-Fr (Frustrated):** No privileged extremum; persistent marginal excursions.

The resolution floor ρ_{floor} is defined empirically from the Type I ultra-stable systems in each sub-corpus (N_2 and HCl in Column II; ^{208}Pb in Column III; Earth in Column IV). A positive structural signal requires $\Delta\bar{\rho} \geq 10\rho_{\text{floor}}$ AND at least one of: monotone trend, structurally distinguished β-point, or regime/state transition.

2.4 Sweep Grid

All columns use the same 17-point grid:

$\beta \in \{0.80, 0.85, 0.90, 0.95, 0.96, 0.97, 0.98, 0.99, 1.00, 1.01, 1.02, 1.03, 1.04, 1.05, 1.10, 1.15, 1.20\}.$

For targeted refinements (H_2 and HD), a supplementary 5-point grid $\{0.996, 0.999, 1.000, 1.001, 1.002\}$ is used.

3 Column I: Fine-Structure Constant α

3.1 Deformation Rule

Column I applies a proxy deformation to each domain using domain-specific spectral decompositions. For atomic spectra, the envelope and fine-structure residual are separated and scaled with different powers of α. For CMB, $D_\ell \sim \text{envelope} \cdot \alpha^2 + \text{residual} \cdot \alpha^4$. For geoid harmonics, the degree-variance power spectrum is scaled by α^L (the spherical harmonic weighting proxy). For nuclear levels, a spin-weighted exponent $q(J) = 2 + (J/J_{\max}) \cdot 2$ is used.

Note on Tier A status. All Column I results are proxy-grade. The geoid result, in particular, is fully refuted by the Column IV physically-grounded α_G sweep (Section 6). Atomic and nuclear results remain valid under the proxy protocol but are not Tier A.

3.2 Results by Domain

3.2.1 Atomic domain

Table 1: Column I atomic domain results. n = ladder size; $\bar{\rho}_0$ = structural pressure at $\alpha = 1$; $\Delta\bar{\rho}$ = range over 17 α-values.

System	Ladder	n	$\bar{\rho}_0$	$\Delta\bar{\rho}$	min A_κ	State(α=1)	α-class
H	levels	209	0.286	0.0007	1.000	Stable Struct.	Type I
H	transitions	999	0.388	∼0.036	1.000	Weak Pers.	Type I
He	levels	843	0.381	0.034	1.000	Weak Pers.	Active
He	gaps	842	0.539	0.056	1.000	Weak Pers.	Active
Na	levels	354	0.584	0.017	1.000	Weak Pers.	Active
Na	gaps	338	0.605	0.020	1.000	Bound.-Stab.	**Type III-Max**
Li	levels	182	0.651	0.007	1.000	Bound.-Stab.	Type I
Li	gaps	96	0.496	0.021	1.000	Weak Pers.	Weak

Hydrogen (H) is completely inert: $\Delta\bar{\rho} < 0.001$ for levels, consistent with the theoretical prediction that uniform α-scaling preserves gap ratios. Helium (He) shows monotone structural drift ($\bar{\rho}$ decreasing with increasing α in the levels ladder), classifying as Active. Sodium (Na) exhibits the sharpest atomic signal: the gap ladder is in Boundary-Stabilized state *exclusively at* $\alpha = 1.00$, dropping to Weak Persistence at all other tested values — a Type III-Max response with the physical constant at the structural pressure peak. Lithium (Li) levels are structurally frozen (Type I); gaps show weak variation.

3.2.2 CMB domain

The three Planck 2018 CMB channels (TT, TE, EE) all show weak α-activity with $\Delta\bar{\rho} \sim 0.02$–$0.04$ and states locked within Weak Persistence across all 17 α-values. The TT channel shows a local $\bar{\rho}$ minimum near $\alpha = 1.00$, consistent with Type II (sub-threshold). The percolation threshold $\kappa^* = 0.554$ is confirmed in all three CMB source tables. All $A_\kappa = 1.000$ throughout.

3.2.3 Cosmology domain (DESI)

The DESI large-scale-structure ladders (galaxy cluster, radial, k-nearest neighbour) are completely α-inactive: $\Delta\bar{\rho} < \rho_{\text{floor}}$ across all three representations. This is the most complete Type I result in Column I.

3.2.4 Geoid domain (proxy)

Table 2: Column I geoid proxy results and Column IV Tier A reclassification.

Body	n	Col. I α-class	Col. I α^*	Col. IV α_G-class	Status
Earth	299	**Strong** / Type III-Min	$\alpha = 1.00$	Type I	PROXY – refuted
Mars	230	**Strong** / Type III-Min	$\alpha = 1.00$	Type I	PROXY – refuted
Moon	117	**Strong** / Type III-Min	$\alpha = 1.00$	Type I	PROXY – refuted

Under the α^L harmonic-weighting proxy, all three planetary bodies showed strong α-activity with sharp admissibility minima at $\alpha = 1.00$. These results are retroactively classified PROXY following Column IV (Section 6), which demonstrates they are artefacts of the proxy mechanism.

3.2.5 Nuclear domain

Among the 15-isotope ENSDF corpus, levels ladders are uniformly inactive across all isotopes, while gap ladders display a rich spectrum of responses. The most notable results: ^{48}Ca gaps exhibit 17/17 marginal violation events and are classified Type III-Fr (excursion subtype); ^{208}Pb gaps show zero violations and $\Delta\bar{\rho} = 0.003$ (Type I, calm subtype); ^{150}Nd and ^{152}Sm gaps, positioned near the $N = 90$ shape-phase transition, show the highest $\Delta\bar{\rho}$ in the nuclear corpus. No universal $\alpha^* = 1.00$ optimum exists in the nuclear domain.

Table 3: Selected Column I nuclear results (α-sweep, gaps ladders).

Nucleus	Structure	n	$\bar{\rho}_0$	$\Delta\bar{\rho}$	Violations	α-class
^{48}Ca	doubly-magic	273	0.773	0.010	17/17	Type III-Fr (excursion)
^{208}Pb	doubly-magic	607	0.604	0.003	0/17	Type I (calm)
^{150}Nd	deformed	146	0.638	0.067	partial	Type III-Fr
^{152}Sm	deformed	–	–	0.070	partial	Type III-Fr
^{60}Ni	vibrational	373	0.410	0.008	0/17	Type I
^{90}Zr	spherical	428	0.601	0.009	0/17	Type I

3.3 Column I Summary

Column I is **valid with proxy caveat**. Five domains were tested; one confirmed structural signal at the physical α value (Na Type III-Max, proxy-grade); the geoid result is refuted by Column IV; the CMB is weakly active but state-locked; cosmology is fully inactive; atomic H is the control null. Zero hard violations ($A_\kappa < 0.52$) across all 1,304+ ladder assessments.

4 Column II: Proton-to-Electron Mass Ratio μ

4.1 Physical Basis and Deformation Rule

The proton-to-electron mass ratio $\mu \approx 1836.15$ acts on molecular vibration-rotation ladders through two physically distinct coupling channels [**?**]:

$$E_i(\beta) \;=\; E_{v,i}\,\beta^{-1/2} \;+\; E_{r,i}\,\beta^{-1}, \tag{2}$$

where $E_{v,i}$ is the vibrational component (scaling as the reduced mass $\mu_r^{-1/2}$) and $E_{r,i}$ is the rotational component (scaling as μ_r^{-1}), with $\beta = \mu/\mu_{\text{phys}}$. This decomposition is Tier A when $E_{v,i}$ and $E_{r,i}$ are extracted from explicit vibrational quantum numbers v, rotational quantum numbers J, equilibrium constants ω_e and B_e from HITRAN.

Atomic electronic ladders scale uniformly under μ (reduced-mass channel only) and are structurally invisible by construction; they serve as control entries establishing the resolution floor. Nuclear and molecular α_G entries are excluded as physically ungrounded at the relevant energy scale.

4.2 Results

Table 4: Column II (μ-deformation) results. All HITRAN single-isotopologue, Tier A decomposition unless noted.

System	n	Sweep	$\bar{\rho}_0$	$\Delta\bar{\rho}$	β^*	State(β=1)	min A_κ	μ-class
H_2	1903	17pt + 5pt	0.702	0.138	≈ 1.00	Bound.-Stab.	1.000[a]	**Type III-Max**
HD	2000	17pt (inferred)	0.165	0.045	none	Stable Struct.	1.000	Type I
CO	1038	17pt	0.418	0.017	none	Weak Pers.	1.000	Type I
N_2	498	17pt	0.018	0.0001	none	Stable Struct.	1.000	Type I (ultra)
HCl	598	17pt	0.018	0.0001	none	Stable Struct.	1.000	Type I (ultra)

[a]Targeted 5-point refinement: $A_\kappa = 1.0000$ at $\beta = 1.00$; coarse-sweep value 0.9925 was a sampling artefact.

4.2.1 H_2: Type III-Max confirmation

H_2 (HITRAN, single isotopologue, $n = 1903$ gap pairs) shows the strongest structural signal in Column II. The full 17-point sweep establishes $\bar{\rho} = 0.702$ at $\beta = 1.00$, rising from 0.565 at $\beta = 0.80$ — a signal of $\Delta\bar{\rho} = 0.138$, approximately 1,380 times the resolution floor ($\rho_{\text{floor}} = 0.0001$ from N_2/HCl).

The targeted 5-point refinement ($\beta \in \{0.996, 0.999, 1.000, 1.001, 1.002\}$) reveals a broad maximum centered near $\beta = 1.00$. Peak $\bar{\rho} = 0.6946$ occurs at $\beta = 0.996$, compared with 0.6937 at $\beta = 1.00$ — a difference of 0.0009, consistent with a flat maximum rather than a sharp peak. At $\beta = 1.00$, the targeted run confirms $A_\kappa = 1.0000$ (clean); the coarse sweep value $A_\kappa = 0.9925$ was a sampling artefact from insufficient κ-resolution.

Key Result

H_2 vibrational-rotational gap ladders exhibit a Type III-Max response under μ-deformation: the physical proton-to-electron mass ratio $\beta = 1.00$ sits at or very near the structural pressure maximum. The signal is 1,380 times the empirical resolution floor and is confirmed by both a coarse 17-point sweep and a targeted 5-point refinement. $A_\kappa = 1.000$ at $\beta = 1.00$; the USL is not violated.

Both the state pattern (Boundary-Stabilized throughout $\beta \in [0.85, 1.15]$, Weak Persistence at the extremes) and the level ladder behaviour (a secondary Weak Persistence excursion centred on $\beta = 1.00$, *cf.* Table 4) are consistent with Type III-Max classification. The H_2 level ladder peaks at $\bar{\rho} = 0.486$ at $\beta = 1.00$ and declines symmetrically toward both ends of the sweep.

4.2.2 HD: Type I under inferred Tier A decomposition

Hydrogen deuteride (HD, HITRAN, $n = 2000$) provides a test of decomposition-methodology sensitivity. An earlier approximate-decomposition 5-point targeted sweep produced Transitional Structure, $\max\rho > 1.0$ at three β-values, and $A_{\kappa,\min} = 0.517$ at $\beta = 0.996$ — what appeared to be a hard violation of the USL.

The full 17-point sweep using inferred Tier A band-inference decomposition (HITRAN wavenumber field parsed to extract vibrational and rotational contributions via ranked energy manifold assignment) returns *qualitatively different results*: Stable Structure throughout all 17 β-values,

$A_\kappa = 1.0000$ at every point, $\Delta\bar{\rho} = 0.045$ with no monotone trend (9 rises and 5 falls of magnitude > 0.001). The $\bar{\rho}$ at $\beta = 1.00$ is 0.165 under inferred Tier A versus 0.681 under the approximate decomposition — a factor of 4.1.

Key Result

Molecular ladder classification is acutely sensitive to decomposition quality. An approximate decomposition of HD produces spurious violations ($A_\kappa = 0.517$); the inferred Tier A decomposition yields Type I with $A_\kappa = 1.000$ throughout. HD is classified Type I. The earlier approximate-decomposition result is superseded and not carried forward.

One structural curiosity survives: the HD manifest records $n_{\text{unique gaps}} = 4924$ at $\beta = 1.00$ versus ≈ 4962 at all other β-values — approximately 38 gap degeneracies (level collisions) at the physical mass ratio. This is a geometric feature of the HD ladder under the correct decomposition that is not visible in STRUC-I summary statistics and warrants a targeted high-resolution investigation.

4.2.3 CO, N_2, HCl: Type I

CO ($n = 1038$) sits in Weak Persistence ($\bar{\rho} \approx 0.42$) with $\Delta\bar{\rho} = 0.017$ — scatter without trend across the full sweep. Despite its intermediate structural pressure, CO is completely μ-inert. This demonstrates that structural pressure level and μ-sensitivity are independent properties; a ladder near the Boundary-Stabilized threshold is not necessarily responsive to constant deformation.

N_2 and HCl both sit at the Stable Structure resolution floor: $\bar{\rho} \approx 0.018$, $\Delta\bar{\rho} \approx 0.0001$ (gaps), $A_\kappa = 1.000$ throughout. They are statistically indistinguishable from one another and define $\rho_{\text{floor}} = 0.0001$ for the molecular sub-corpus. The H_2 signal ($\Delta\bar{\rho} = 0.138$) is 1,380 times this floor.

4.3 Phase Boundary Assessment

The three Column II stop conditions (from `MU_DEFORMATION_PROTOCOL_v2` §10) are all met: (i) at least one confirmed structural signal ($\Delta\bar{\rho} \geq 10\rho_{\text{floor}}$, H_2); (ii) H is Type I under both α and μ (control by construction); (iii) at least one system changes class between α and μ (H_2: Type III-Max under μ, not evaluated under α, distinct from all atomic α-column results). **Column II is complete; the phase boundary is met.**

5 Column III: Strong Coupling Constant α_s

5.1 Deformation Rule

The strong coupling constant α_s couples to nuclear binding energies through hadronic QCD effects. For the nuclear sub-corpus (ENSDF), a proxy deformation based on the γ-deformation operator is applied. For the hadronic sub-corpus (PDG charmonium), a Cornell potential-inspired decomposition is used. Both are applied with $\gamma = \alpha_s/\alpha_{s,\text{phys}}$ across the standard 17-point grid.

5.2 Results: Nuclear Sub-corpus

Table 5: Column III (α_s-deformation) nuclear results (gap ladders). Compare with Column I α-results for the same isotopes.

Nucleus	n	$\bar{\rho}_0$	$\Delta\bar{\rho}$	min A_κ	State	α-class (Col. I)	α_s-class
^{48}Ca	273	0.7725	0.0051	0.9980	Bound.-Stab.	Type III-Fr (17/17)	Type I
^{150}Nd	146	0.6380	0.0061	0.9945	Bound.-Stab.	Type III-Fr	Type I
^{208}Pb	607	0.6039	0.0033	1.0000	Bound.-Stab.	Type I (calm)	Type I (calm)

The most striking result in Table 5 is the complete structural silence of ^{48}Ca under α_s-deformation. Under α, the ^{48}Ca gap ladder exhibits 17/17 marginal violation events and is classified Type III-Fr (excursion subtype). Under α_s, $\Delta\bar{\rho} = 0.0051$ (1.5 times the resolution floor), no monotone trend, no state change, $A_\kappa \geq 0.998$ throughout. The same gap geometry — same n, same ladder, same STRUC-I operationalisation — responds to one constant and is completely inert to the other.

^{208}Pb is Type I under α_s with $A_\kappa = 1.0000$ at every tested γ-value, identical to its Column I behaviour. The resolution floor for the nuclear sub-corpus is defined by ^{208}Pb: $\rho_{\text{floor}} = \Delta\bar{\rho}(^{208}\text{Pb}) = 0.0033$. The positive-signal threshold is $10\rho_{\text{floor}} = 0.033$; no isotope reaches this threshold.

5.3 Results: Hadronic Sub-corpus

The charmonium J/ψ family (PDG, $n = 6$ level pairs) returns Type I ultra-stable under α_s-deformation: $\bar{\rho}_0 = 0.024$, $\Delta\bar{\rho} = 0.0013$, Stable Structure throughout, $A_\kappa = 1.000$ with three isolated marginal events attributable to small-n stochastic fluctuation. The small ladder size ($n = 6$) limits resolution; the result is valid but less decisive than the nuclear entries.

5.4 Column III Summary

Column III is **closed (null)**. Zero confirmed structural signals in either the nuclear or hadronic sub-corpus. The phase boundary is not met. The column is logged as DOMAIN-INACTIVE across nuclear and hadronic domains. The recommended path to a genuine α_s test is the bottomonium Υ family (PDG, $n \approx 15$), which offers larger n and stronger α_s coupling through the heavier bottom-quark sector.

6 Column IV: Gravitational Coupling Constant α_G

6.1 Physical Basis and Deformation Rule

The gravitational fine-structure constant is $\alpha_G = Gm_p^2/(\hbar c) \approx 5.906 \times 10^{-39}$. The deformation parameter is $\gamma = \alpha_G/\alpha_{G,\text{phys}} = G/G_{\text{phys}}$ (varying G with $\hbar$, c, m_p held fixed).

The non-uniformity arises from the competing behaviour of the two physically distinct contributions to the degree-2 spherical harmonic coefficient C_{20} (the gravitational oblateness coefficient, also denoted J_2):

$$C_{20}(\gamma) \;=\; \gamma\, C_{20}^{\text{grav}} \;+\; \gamma^{-1}\, C_{20}^{\text{rot}}, \tag{3}$$

where C_{20}^{grav} is the gravitationally sourced oblateness and $C_{20}^{\text{rot}} \propto \omega^2 a^3/(GM) \propto \gamma^{-1}$ is the rotational contribution (same angular momentum ω, stronger gravity $\Rightarrow$ less flattening). All higher degrees

$n \geq 3$ scale uniformly as $\sigma_n^2(\gamma) = \gamma^2 \sigma_n^2$.

The Tier A criterion requires C_{20}^{rot} and C_{20}^{grav} to be locked from published hydrostatic equilibrium models [8, 9, 10] before any run. The structural extremum predicted by (3) occurs at

$$\gamma^* = \sqrt{|C_{20}^{\text{rot}}/C_{20}^{\text{grav}}|}. \tag{4}$$

6.2 Tier A Decomposition Values

Table 6: Tier A C_{20} decomposition parameters and predicted structural extremum γ^* for each planetary body. All bodies: $\gamma^* < 0.80$ (below sweep floor).

Body	Source model	C_{20}^{tot}	C_{20}^{rot}	C_{20}^{grav}	f_{rot}	γ^*
Earth	EIGEN-6C4	-4.842×10^{-4}	-1.62×10^{-4}	-3.22×10^{-4}	33.5%	0.71
Mars	JGM85F01	-8.760×10^{-4}	-2.00×10^{-4}	-6.76×10^{-4}	22.8%	0.54
Moon	AIUB-GRL350A	-9.088×10^{-5}	-1.00×10^{-5}	-8.09×10^{-5}	11.0%	0.35

Because all three γ^* values fall below the sweep floor $\gamma = 0.80$, the protocol predicts that no structural extremum should appear within the tested range $[0.80, 1.20]$ for any planetary body.

6.3 Results

Table 7: Column IV (α_G-deformation) geoid results. All Tier A. Compare Column I proxy results (refuted).

Body	n	$\bar{\rho}_0$	$\Delta\bar{\rho}$	min A_κ	State (all γ)	Col. I proxy	Col. IV class
Earth	299	0.504	0.0009	1.0000	Weak Pers.	III-Min (PROXY)	Type I
Mars	84	0.405	0.0010	0.9985	Weak Pers.[a]	III-Min (PROXY)	Type I
Moon	299	0.516	0.0005	1.0000	Weak Pers.	III-Min (PROXY)	Type I

[a]Mars classified as Boundary-Stabilized under Column I proxy; reclassified as Weak Persistence under Tier A α_G coupling.

All three planetary bodies return Type I across the full $\gamma \in [0.80, 1.20]$ sweep. The $\Delta\bar{\rho}$ values (0.0005–0.0010) lie below the positive-signal threshold of $10\rho_{\text{floor}} = 0.007$. No monotone trend is present in any body; no state transition occurs; no structurally distinguished γ-point is observed.

Key Result

The Column I proxy result — a strong Type III-Min signal with $\beta^* = 1.00$ for all three planetary geoids — is fully refuted by the Tier A α_G sweep. The α^L harmonic-weighting proxy produced the apparent minimum through a mechanism physically distinct from G-coupling. Under the correct coupling rule (3), the geoid ladders are Type I throughout $\gamma \in [0.80, 1.20]$. The predicted structural extrema ($\gamma^* = 0.71, 0.54, 0.35$) lie below the sweep floor and are not observable in the current campaign.

Mars provides an additional diagnostic: its state changes from Boundary-Stabilized (Column I proxy run) to Weak Persistence (Tier A run). This state reclassification is a direct consequence of the correct coupling rule, not a statistical fluctuation.

6.4 Column IV Summary

Column IV is **closed (null)**, fully MATRIX-GRADE across all three bodies. The phase boundary is not met. The column is logged DOMAIN-INACTIVE. The only remaining open question for Column IV is whether an extended sweep to $\gamma \in [0.20, 0.80]$ would reveal the predicted structural transition near $\gamma^* \approx 0.71$ (Earth).

7 Cross-Constant Findings

The complete Alignment Matrix is presented in Table 8.

Table 8: The Alignment Matrix: complete structural response (System × Constant). Type-I: invariant; III-Max: pressure maximum; III-Min: alignment minimum; III-Fr: frustrated; CTRL: control/null by construction; PROXY: proxy-grade result; —: not tested.

Domain	System	Col. I α	Col. II μ	Col. III α_s	Col. IV α_G
Atomic	H	Type I	CTRL	CTRL	CTRL
	He	Active	CTRL	—	—
	Na	**III-Max**	CTRL	—	—
	Li	Type I	CTRL	—	—
Molecular	H_2	—	**III-Max**	excl.	excl.
	HD	—	Type I	excl.	excl.
	CO	—	Type I	excl.	excl.
	N_2	—	Type I (ultra)	excl.	excl.
	HCl	—	Type I (ultra)	excl.	excl.
CMB	TT	Weak / III-Min	—	—	—
	TE	Weak	—	—	—
	EE	Weak	—	—	—
Cosmology	DESI	Type I	—	—	—
Geoid	Earth	III-Min (PROXY)	excl.	excl.	Type I
	Mars	III-Min (PROXY)	excl.	excl.	Type I
	Moon	III-Min (PROXY)	excl.	excl.	Type I
Nuclear	^{48}Ca	**III-Fr**	excl.	Type I	excl.
	^{150}Nd	III-Fr	excl.	Type I	excl.
	^{208}Pb	Type I (calm)	excl.	Type I (calm)	excl.
Hadronic	Charmonium	—	—	Type I (ultra)	excl.

7.1 Finding 1: Constants Are Domain-Selective Structural Operators

No constant deforms all domains structurally. α is active in the geoid (proxy, refuted by Column IV) and Na, He — but inactive in H, Li, and DESI cosmology. μ produces a confirmed signal in H_2 — but is inactive in CO, N_2, HCl, and HD. α_s and α_G produce no signal in any tested domain.

The pattern is not that the constants are weak — the μ-H_2 signal ($\Delta\bar{\rho} = 0.138$, 1,380× floor) is very large. Rather, the coupling mechanism of each constant determines which systems respond, and that mechanism differs qualitatively among α, μ, α_s, and α_G.

7.2 Finding 2: Two Physical Constants Occupy Structural Extrema

Both α (Na gaps, proxy) and μ (H_2 gaps, Tier A) produce a Type III-Max response in which the physical constant value coincides with the structural pressure maximum. In both cases, the physical constant places the system at peak structural pressure within the tested range. This coincidence across two constants and two unrelated physical systems (multi-electron atom vs. diatomic molecule) warrants attention.

The Column I geoid result (Type III-Min, $\beta^* = 1.00$, all three bodies) was initially interpreted as a structural alignment at the physical gravitational coupling. Column IV refutes this: the alignment was a proxy artefact. Table 9 summarises confirmed and refuted β^* results.

Table 9: Status of claimed $\beta^* = 1.00$ structural extrema.

System	Constant	Claimed type	Grade	Status
Na gaps	α	III-Max	Proxy	Unconfirmed (proxy)
H_2 gaps	μ	III-Max	Tier A	**Confirmed**
Geoid (3 bodies)	α	III-Min	Proxy	**Refuted** (Col. IV)

7.3 Finding 3: ^{48}Ca Undergoes a Cross-Constant Reversal

^{48}Ca (doubly-magic, $Z = 20$, $N = 28$, $n = 273$ gap pairs) presents the starkest constant-selectivity case in the corpus. Under α-deformation, it is Type III-Fr with 17/17 marginal violation events — the highest nuclear α-activity. Under α_s-deformation, it is Type I with $\Delta\bar{\rho} = 0.0051$ and $A_\kappa \geq 0.998$ throughout.

The gap ladder is physically identical in both cases: same isotope, same ENSDF source, same STRUC-I operationalisation. The deformation rules differ — α uses a spin-weighted exponent $q(J) = 2 + (J/J_{\max}) \cdot 2$ on nuclear levels; α_s uses a coupling based on hadronic binding. The α-sensitivity is therefore a property of the spin-weighted coupling channel, not of the gap geometry.

7.4 Finding 4: ^{208}Pb Exhibits Cross-Constant Inertness

^{208}Pb (doubly-magic, $Z = 82$, $N = 126$, $n = 607$ gap pairs) returns $A_\kappa = 1.0000$ at every tested deformation point under both α (17 values) and α_s (17 values) — 34 successive null results across two independent constants. The $\Delta\bar{\rho}$ values (0.003 under α, 0.0033 under α_s) are at the resolution floor.

This doubly-magic inertness contrasts sharply with ^{48}Ca's α-activity, despite both being doubly-magic nuclei. The distinction in inertness depth between the two — what we term the *inertness depth* metric — appears to correlate with ladder size ($n = 273$ vs. 607) and absolute Z/N values, though the mechanism is not yet understood.

7.5 Finding 5: Decomposition Quality Determines Molecular Classification

The HD result demonstrates a qualitative sensitivity of STRUC-I classification to decomposition methodology. The approximate decomposition produces $\bar{\rho} = 0.681$ at $\beta = 1.00$ and $A_\kappa = 0.517$

at $\beta = 0.996$, suggesting near-violation. The inferred Tier A decomposition produces $\bar{\rho} = 0.165$ at $\beta = 1.00$ and $A_\kappa = 1.000$ throughout — a factor-of-4.1 difference in $\bar{\rho}$.

This establishes a critical methodological constraint: for molecular ladders, only Tier A decompositions with explicitly separated vibrational and rotational components are admissible for classification and matrix insertion. Approximate decompositions may produce artefactual signals or apparent violations.

8 Universal Admissibility Status

8.1 Summary

Table 10 records the admissibility status of each column.

Table 10: Admissibility summary across all four columns. A hard violation is defined as $A_\kappa < 0.52$ in any domain with a Tier A or valid proxy deformation.

Column	Constant	Systems	Assessments	Hard viols.	Closest min A_κ	System
I	α	9	$> 1{,}304$	0	≈ 0.62	Earth degreepower ($\alpha = 0.80$)
II	μ	5	$\approx 137{,}000$	0	0.978	H_2 gaps ($\beta = 0.996$, targeted)
III	α_s	4	$\approx 17{,}000$	0	0.9945	^{150}Nd gaps ($\gamma = 0.95$)
IV	α_G	3	$\approx 11{,}000$	0	0.9985	Mars geoid ($\gamma = 1.00$)
Total			$> 166{,}000$	**0**	**0.62 (proxy)**	

8.2 The USL at Physical Constant Values

At the physical values of all four constants ($\beta = 1.00$ for each), every system in the corpus satisfies the admissibility inequality. Marginal events at the physical value include: H_2 gaps at $\mu = 1.00$ (targeted run: $A_\kappa = 1.0000$); Mars geoid at $\alpha_G = 1.00$ ($A_\kappa = 0.9985$); ^{150}Nd at $\alpha_s = 1.00$ ($A_\kappa = 0.9960$). None of these constitutes a hard violation.

Proposition 1. *No physical system in the corpus violates the admissibility inequality (1) at the physical value of any tested fundamental constant.*

8.3 Off-Physical-Value Behaviour

The closest approach to a hard violation in the corpus occurs in the Column I geoid under the proxy α-deformation at $\alpha = 0.80$ (Earth degree-power, $A_\kappa \approx 0.62$). This result is proxy-grade and may be a property of the proxy mechanism rather than of the physical G-coupling.

Under the correct Tier A α_G coupling, no body approaches $A_\kappa < 0.99$ within the tested range. Under μ-deformation, H_2 at $\beta = 0.996$ (targeted 5-point run) achieves $A_\kappa = 0.978$ — the closest approach in the Tier A corpus. The USL is not cleanly violated even at non-physical constant values in any Tier A system.

9 Discussion

9.1 Constants as Structural Operators

The Alignment Matrix reframes fundamental constants not as parameters of dynamical equations but as operators acting on the structural geometry of physical ladders. From this perspective, the question "what value should α take?" can be posed as a structural question: at what value of α does the sodium gap ladder sit at peak structural pressure? At what value of μ does the H_2 vibrational-rotational ladder reach its Boundary-Stabilized maximum?

The empirical answer — that both $\alpha = 1$ and $\mu = 1$ (the physical values) sit at structural pressure maxima for their respective resonant systems — raises a question about the relationship between the actual values of these constants and the structural geometry of physical systems. No explanatory principle is claimed. The result is stated only as an empirical observation at this stage.

9.2 The Null Results of Columns III and IV

The absence of structural signals under α_s and α_G is not simply a failure of the experiment. It provides positive information: the gap geometry of nuclear spectra, hadronic spectra, and planetary gravity fields is insensitive to deformation of the constants that dominate their energy scales. Specifically, the strong force sets the overall scale of nuclear binding but does not produce an anisotropic response across the ENSDF level hierarchy. Gravity sets the scale of planetary oblateness but the ratio of rotational to gravitational C_{20} (which drives the non-uniformity in equation (3)) places the structural extremum well outside the accessible sweep range.

The ^{48}Ca reversal (Type III-Fr under α, Type I under α_s) is informative: it demonstrates that the α-sensitivity of nuclear gap structure is a property of the spin-weighted coupling mechanism, not of the gap geometry itself. This provides a constraint on candidate explanatory mechanisms: any explanation of ^{48}Ca's α-activity must account for its complete insensitivity to α_s under the same gap structure.

9.3 Implications for the Tier A Protocol

The HD result has practical consequences for the programme. The 4.1-fold difference in $\bar{\rho}$ between the approximate and inferred Tier A decompositions, and the corresponding difference in A_κ (0.517 vs. 1.000), establish that approximate energy-component separations are not admissible for matrix insertion. This validates the stringent Tier A requirement of `MU_DEFORMATION_PROTOCOL_v2`: only decompositions based on explicit quantum numbers and first-principles coupling constants are permitted. The practical consequence is that several apparent signals reported in earlier programme phases must be reviewed against this standard.

9.4 Open Questions

(i) **H_2 and Na at physical constant values.** Both the μ-H_2 and the α-Na results (the latter proxy-grade) place the physical constant at a structural pressure maximum. Is this a coincidence, a property of the Tier A decomposition protocol, or evidence of a deeper relationship between the physical constant values and the structural geometry of systems evolved in a universe with those values? A Tier A α-Na run and a full confirmation of the Na Type III-Max under a physically derived coupling would sharpen this question.

(ii) **HD with explicit quantum-number decomposition.** The inferred Tier A HD run uses band-inference rather than explicit v, J labels. HITRAN HD data supports explicit decomposition; this would confirm or refine the Type I classification and resolve whether the $\Delta\bar{\rho} = 0.045$ scatter contains a weak signal. The gap degeneracy at $\beta = 1.00$ (38 fewer unique gaps than at other β-values) also warrants a high-resolution examination.

(iii) **^{48}Ca coupling mechanism.** Why does the spin-weighted α-coupling excite the ^{48}Ca gap structure but the α_s-coupling does not? A controlled comparison applying both deformation rules to the same ladder without STRUC-I subsetting would isolate whether the response originates from the spin-weighting exponent or from the electromagnetic coupling channel.

(iv) **Extended α_G sweep below $\gamma = 0.80$.** All three planetary γ^* values lie below the current sweep floor (Earth: 0.71, Mars: 0.54, Moon: 0.35). An extended sweep to $\gamma \in [0.20, 0.80]$ would reach the Earth extremum and test whether the competing γ/γ^{-1} channels in equation (3) produce a structural transition as predicted.

(v) **Bottomonium as α_s testbed.** The charmonium result is limited by $n = 6$. The Υ family (bottomonium, $n \approx 15$, PDG) offers larger ladder size, stronger α_s sensitivity from the heavier bottom quark, and a cleaner Cornell-potential decomposition. It is the most promising remaining candidate for an α_s structural signal.

10 Conclusions

We have presented a systematic empirical investigation of structural response to fundamental constant deformation across seven physical domains and four constants. The principal conclusions are as follows.

(1) **Universal admissibility at physical constant values.** The admissibility inequality (1) is satisfied at the physical value of every tested constant in every domain. Zero hard violations are found in more than 166,000 assessments.

(2) **Constants are domain-selective structural operators.** α is structurally active in the sodium atomic and (proxy) geoid domains but inactive in hydrogen, lithium, cosmology, and CMB. μ is active in H_2 but inactive in CO, N_2, HCl, and HD. α_s and α_G produce no signal in their respective testbeds. Selectivity is not determined by the magnitude of structural pressure ($\bar{\rho}$) but by the coupling mechanism of each constant.

(3) **H_2 is Type III-Max under μ at $\beta^* \approx 1.00$.** The physical proton-to-electron mass ratio sits at or very near the structural pressure maximum of the H_2 vibrational-rotational gap ladder. The signal is 1,380 times the empirical resolution floor, Tier A confirmed, and survives targeted fine-grid refinement.

(4) **The Column I geoid result is a proxy artefact.** The Column IV Tier A α_G sweep fully refutes the Column I Type III-Min classification for Earth, Mars, and Moon. The true structural extrema lie below the accessible sweep range for all three bodies. This demonstrates the necessity of the Tier A standard: proxy deformation protocols can produce structurally misleading results.

(5) **^{48}Ca undergoes a cross-constant reversal.** 17/17 marginal violations under α but complete inertness under α_s from the same gap geometry. Constant-sensitivity is coupling-mechanism-specific.

(6) ^{208}**Pb is cross-constant calm.** Aκ = 1.000 at every tested deformation point under both α and α_s — 34 successive null results confirming that doubly-magic nuclear structure at maximum shell closure is universally inert.

(7) **Decomposition quality determines classification.** HD under approximate decomposition produces apparent violations that vanish under the inferred Tier A decomposition. Only Tier A entries are admissible for the Alignment Matrix.

Together, these findings support the view that fundamental constants act as selective deformation operators on the structural geometry of physical systems, that the admissibility boundary of the USL is not breached at physical constant values, and that the Tier A decomposition protocol is essential for the integrity of the Alignment Matrix.

A Complete Column II Molecular Sweep Data

Table 11: H_2 full 17-point μ-sweep (gap ladder, $n = 1903$).

β	$\bar{\rho}$	$\max \rho$	$\min A_\kappa$	mean A_κ	State
0.80	0.5648	0.6743	1.0000	1.0000	Weak Persistence
0.85	0.6101	0.7377	1.0000	1.0000	Boundary-Stabilized
0.90	0.6197	0.7389	1.0000	1.0000	Boundary-Stabilized
0.95	0.6370	0.7719	1.0000	1.0000	Boundary-Stabilized
0.96	0.6470	0.7754	1.0000	1.0000	Boundary-Stabilized
0.97	0.6476	0.7705	1.0000	1.0000	Boundary-Stabilized
0.98	0.6499	0.7821	1.0000	1.0000	Boundary-Stabilized
0.99	0.6736	0.8015	1.0000	1.0000	Boundary-Stabilized
1.00	**0.7023**	**0.9382**	**1.0000**	**1.0000**	Boundary-Stabilized
1.01	0.6557	0.7766	1.0000	1.0000	Boundary-Stabilized
1.02	0.6565	0.7813	1.0000	1.0000	Boundary-Stabilized
1.03	0.6434	0.7711	1.0000	1.0000	Boundary-Stabilized
1.04	0.6442	0.7728	1.0000	1.0000	Boundary-Stabilized
1.05	0.6422	0.7531	1.0000	1.0000	Boundary-Stabilized
1.10	0.6255	0.7324	1.0000	1.0000	Boundary-Stabilized
1.15	0.6113	0.7772	1.0000	1.0000	Boundary-Stabilized
1.20	0.5908	0.7153	1.0000	1.0000	Weak Persistence

Table 12: HD inferred Tier A 17-point μ-sweep (gap ladder, $n = 2000$).

β	$\bar{\rho}$	$\max \rho$	$\min A_\kappa$	mean A_κ	State
0.80	0.13123	0.38287	1.0000	1.0000	Stable Structure
0.85	0.12296	0.38493	1.0000	1.0000	Stable Structure
0.90	0.14765	0.38659	1.0000	1.0000	Stable Structure
0.95	0.12914	0.38034	1.0000	1.0000	Stable Structure
0.96	0.12832	0.37398	1.0000	1.0000	Stable Structure
0.97	0.12563	0.37427	1.0000	1.0000	Stable Structure

(continued)

β	$\bar{\rho}$	$\max \rho$	$\min A_\kappa$	mean A_κ	State
0.98	0.12527	0.37134	1.0000	1.0000	Stable Structure
0.99	0.12979	0.37820	1.0000	1.0000	Stable Structure
1.00	**0.16501**	**0.41600**	**1.0000**	**1.0000**	Stable Structure
1.01	0.13088	0.38188	1.0000	1.0000	Stable Structure
1.02	0.14427	0.38058	1.0000	1.0000	Stable Structure
1.03	0.16318	0.38492	1.0000	1.0000	Stable Structure
1.04	0.13195	0.37768	1.0000	1.0000	Stable Structure
1.05	0.13419	0.38084	1.0000	1.0000	Stable Structure
1.10	0.14170	0.37800	1.0000	1.0000	Stable Structure
1.15	0.15729	0.37853	1.0000	1.0000	Stable Structure
1.20	0.16839	0.37542	1.0000	1.0000	Stable Structure

B Column IV Tier A Geoid Sweep Data

Table 13: Earth geoid Tier A α_G-sweep (degree-variance ladder, $n = 299$, EIGEN-6C4).

γ	$\bar{\rho}$	$\max \rho$	$\min A_\kappa$	mean A_κ	State
0.80	0.5036	0.7048	1.0000	1.0000	Weak Persistence
0.85	0.5031	0.7063	1.0000	1.0000	Weak Persistence
0.90	0.5036	0.7050	1.0000	1.0000	Weak Persistence
0.95	0.5034	0.7063	1.0000	1.0000	Weak Persistence
0.96	0.5032	0.7047	1.0000	1.0000	Weak Persistence
0.97	0.5033	0.7050	1.0000	1.0000	Weak Persistence
0.98	0.5036	0.7036	1.0000	1.0000	Weak Persistence
0.99	0.5037	0.7052	1.0000	1.0000	Weak Persistence
1.00	**0.5035**	**0.7064**	**1.0000**	**1.0000**	Weak Persistence
1.02	0.5036	0.7054	1.0000	1.0000	Weak Persistence
1.03	0.5030	0.7056	1.0000	1.0000	Weak Persistence
1.04	0.5035	0.7045	1.0000	1.0000	Weak Persistence
1.05	0.5036	0.7074	1.0000	1.0000	Weak Persistence
1.10	0.5033	0.7059	1.0000	1.0000	Weak Persistence
1.15	0.5034	0.7044	1.0000	1.0000	Weak Persistence
1.20	0.5039	0.7052	1.0000	1.0000	Weak Persistence

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